Section 5.4

The Fundamental Theorem of Calculus I

(1) The Fundamental Theorem of Calculus Part I

(2) The Indefinite Integral

(3) The Net Change Theorem

The Fundamental Theorem of Calculus I (FTC-1)

If f is continuous on the interval $[a, b]$, then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

where F is **any** antiderivative of f, that is, $F' = f$.

Example 1: Evaluate $\int_1^3 \left(x^2 - 6\right)\,dx$.

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Example 2: Evaluate $\int_0^{\pi/2} (2\cos(x) - 4x)dx$.

FTC-1: Alternative Formulations

We frequently use the notation $F(x)\Big|_a^b$ to stand for $F(b) - F(a)$, so that FTC-1 can be rewritten as

$$\int_a^b f(x)\,dx = F(x)\Big|_a^b$$

FTC-1 says that definite and indefinite integrals are related as follows:

$$\int_a^b f(x)\,dx = \int f(x)\,dx\,\Big|_a^b$$

Example 3: Evaluate $\displaystyle\int_1^4 \frac{5x^2 - \sqrt{x} - 3}{\sqrt{x}}\,dx.$

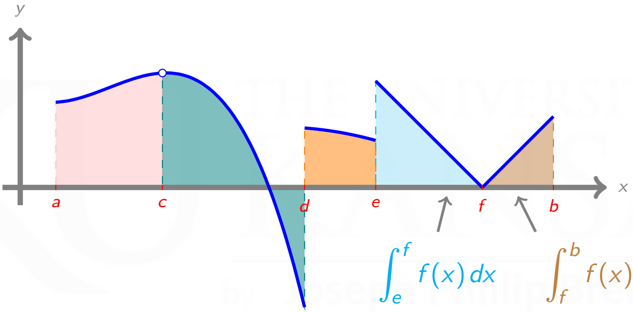
Here is another way of writing FTC-1:

$$\int_a^b F'(x)\,dx \;=\; F(x)\Big|_a^b \;=\; F(b) - F(a).$$

This version of FTC is often referred to as the **Net Change Theorem**, because it says that the integral of $F'(x)$ — that is, the integral of the rate of change of $F(x)$ — is the net change in F.

Example 4: Water is leaking out of the bottom of a storage tank. The rate of flow at time t (in minutes) is $r(t) = 200 - 8t$ (in L/min). How much water is lost between $t = 5$ and $t = 20$?

The statement of FTC-1 applies only to continuous functions, but in fact it can be used to integrate functions whose only discontinuities are a **finite** number of holes or jumps. For example:



$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^d f(x)\,dx + \int_d^e f(x)\,dx + \int_e^f f(x)\,dx + \int_f^b f(x)\,dx$$

and each the four integrals on the right can be evaluated using FTC.

Warning: FTC-1 cannot be used if $f(x)$ has an infinite discontinuity (vertical asymptote). We will explore this further in MATH 126.

Example 5: The velocity function (in meters per second) for a particle moving along a line is

$$v(t) = 3t - 8$$

(i) Find the displacement of the object from time $t = 0$ to time $t = 4$.
(ii) Find the total distance traveled from time $t = 0$ to time $t = 4$.